Comparative Analysis of Computational Thinking in Mathematics Education: A Nordic Discussion

Morten Misfeldt[1], Andreas Tamborg[1], Cecilia Kilhamn[2], and Ola Helenius[2].

[1]University of Copenhagen, Denmark; misfeldt@ind.ku.dk, andreas_tamborg@ind.ku.dk.

[2]University of Gothenburg; cecilia.kilhamn@ped.gu.se, ola.helenius@ncm.gu.se.

This working group report examines the integration of Computational Thinking (CT) in mathematics education across Nordic countries, highlighting variations due to national policies and educational structures. We explain the approaches in Denmark, Sweden, Norway, Finland, Iceland, and Estonia and discuss how to balance mathematical learning with CT. Furthermore, we propose to use a praxeological framework to scaffold the comparison. The findings underscore the need for further research to support successful CT integration in mathematics curricula across diverse educational contexts.

Computational Thinking in the school systems

The concept of computational thinking (CT) has gained prominence in mathematics education research and curriculum policies in recent years (Bocconi et al. 2022). There are many definitions of CT that each highlight different aspects, but a common denominator is that they refer to the thought processes involved in formulating problems so their solutions can be represented as computational steps and algorithms (see e.g. Wing, 2006). Integrating CT into mathematics classrooms is a valuable approach to enhancing learning experiences (Misfeldt & Ejsing-Duun 2015). However, the manner of this integration and its implications vary across countries. This discussion group sought to explore these variations, specifically focusing on the Nordic countries.

A significant challenge in comparing CT integration across countries is the influence of diverse national school policies. These policies, sometimes opaque or inconsistent, affect how CT is implemented and understood in various educational settings (Tamborg, 2022; Bocconi et al. 2022). Furthermore, while the foundations of CT are based on both mathematics and computer science, there is an important question regarding student perception. How do students interpret these concepts, especially when they might not differentiate between the two disciplines? This can lead to challenges such as terminological conflations, since terms like 'problem-solving' sometimes hold different meanings and implications in mathematics compared to computer sciences or other situations where certain symbols, while seemingly similar, might not align across disciplines (such as the % which refers percentage in mathematics and modulo operator in computer science) (Bråting & Kilhamn, 2021).

In the discussion group, participants examined the relationship between how CT was presented in curriculum policies and its representation in educational research. The goal was to understand the challenges and opportunities this presented for the effective integration of CT in mathematics education. Participants were invited to contribute with their insights and experiences as they collectively navigated the complexities of CT in the global educational landscape. In particular they were encouraged to bring short presentations on local situations in different school systems. These presentations, along with inputs from the participants, were used to discuss questions about how and

why integration between programming and mathematics was done in primary and secondary schools in different countries in the Nordic region.

Workshop scope

The workshop began with a round of presentations on the local situations regarding CT and mathematics in primary and lower secondary schools. Following this, participants discussed several focal points for comparison, drawing on investigations and frameworks such as those by Tamborg et al. (2023) and Helenius and Misfeldt (2021). Among other things, these focal points regarded what content from the two disciplines to connect and in what interdisciplinary configuration. Based on these inputs, the group collectively addressed questions about what it means to integrate CT into mathematics teaching, how this integration varies across different Nordic school systems, and how factors like national culture and specific implementation decisions influenced the outcomes of integrating CT into mathematics. This was done through a combination of small group work and plenary discussion.

The situations in Sweden and Denmark

The initial presentations showed the different routes that Denmark and Sweden had taken to the integration of digital competence and technology comprehension (TC).

Although there is potential for meaningful interdisciplinary connections between mathematics and CT, evidence of successful integration remains limited. While many theoretical frameworks suggest how integration should work (e.g. Weintrop et. Al. 2016; Kallia et al. 2021), there are few concrete examples where it demonstrably does. A key challenge lies in conceptualizing mathematics and CT as complementary disciplines, striking a balance between their integration, interaction, and respective weights in the curriculum. Although CT is not explicitly mentioned in the Swedish curriculum and only is a sub-component of the much wider Danish Technology Comprehension (TC) curriculum, the curricula revisions in the two contexts address content related to CT.

Insights from a Danish pilot project on Technology Comprehension (TC) (2018-2021), implemented within existing subjects, reveal significant challenges. In Denmark, TC was implemented in a pilot project that tested TC asa subject in its own right and as integrated in existing subject, here among mathematics (Børne- og Undervisningsministeriet, 2018). The integration into mathematics happened by adding new parallel TC competencies, knowledge and skills to the existing mathematical content . Two-thirds of participating teachers found it necessary to override existing subject content to incorporate TC (Børne- og undervisningsministeriet, 2020). There was a clear demand for authentic teaching materials rooted in real-world contexts, though, notably, several schools applied to extend the pilot, signalling sustained interest. The current direction for TC in Denmark involves embedding it within existing subjects, yet without clear guidelines on what, how, or why this should happen. This situation underscores the need to actively listen to and address the experiences and requests of schools, fostering a proactive, rather than reactive, approach that moves beyond awaiting policy decisions. Furthermore, research on issues such as techno-critical mathematics education (Jankvist et al. 2023) highlights the importance of embedding TC authentically within mathematics education, enriching students' understanding of both disciplines in a real-world context.

In the 2022 Swedish curriculum revision concerned the broader concept of digital competence, which all subjects were revised to address, but which implied different modification in the indidual subjects. In mathematics, programming became embedded within the core content of algebra (Skolverket, 2018). This begins as early as Grades 1-3, where students learn about constructing, describing, and following unambiguous step-by-step instructions as a foundation for programming, including the use of symbols. By Grades 4-6, students advance to creating and applying algorithms in visual programming environments, while Grades 7-9 introduce algorithm development, testing, and refinement in both visual and text-based programming environments.

The Swedish national curriculum of 2018, closely mirrored by the 2022 version, left much open to interpretation and required teachers to define its practical meaning. In essence, terms like "stepwise instructions" and "algorithms" were introduced, alongside requirements for using various programming environments. However, teachers across the country found themselves struggling — what exactly were they supposed to teach, and what were students supposed to learn? With limited guidance and many teachers lacking a background in programming, this posed significant challenges.

The other Nordic Countries

Countries participating in the discussion	Programming included in K-9?	Programming: Disciplinary affinity and relation to specific topics
Denmark	Tested since 2018. Forward (from 2025) it is an elective in grade 8. And a topic to be dealt with in other topics.	Several disciplines including mathematic, and as its own topic.
Sweden	Yes from 2018	Related to mathematics and algebra
Norway	Yes from 2020	Related to mathematics and problem solving
Finland	Yes	Related to mathematics and later an individual topic
Estonia	Programming is offered as an elective	No relation to mathematics – moving away from ICT in mathematics education towards paper and pencil math.
Iceland	No real decisions but a lot of stakeholders are pushing to make programming part of the curriculum.	Both related to mathematics and as an independent topic.

Table 1: An overview of the different countries approaches to programming in relation to school mathematics

Addressing programming through the lens of praxeology

When comparing and discussing the different ways that programming has been introduced into schools and especially into mathematics education, we adopted the notion of praxeology. A praxeology is a way of expressing expertise in terms of both knowing about and area and knowing what to do. It is described by Chevallard (2019), is composed of a *praxis block* (know-how) and a *logos block* (know-why). For programming in mathematics, the praxis block could include practical skills like data handling, statistics, proofs, modelling, sequencing, and coding in digital environments, which encompasses tasks such as tinkering, remixing, and algorithmic techniques. The logos block, on the other hand, refers to understanding the reasoning behind these tasks—why specific techniques are used for particular tasks, and why programming relies on logically constructed, unambiguous steps to produce reliable and repeatable outcomes.

One of the problems that we see all over the Nordic countries is that in the context of programming in the curriculum, this praxeology is not directly and thoroughly provided to teachers (Helenius & Misfeldt, 2021). Consequently, as programming was transposed into the school setting, teachers needed to adapt the knowledge of both praxis (techniques and practical tasks) and logos (the underlying purpose and reasoning). However, without a clear epistemological reference model to guide this adaptation, teachers were left with significant interpretive work, needing to translate and contextualize programming knowledge for mathematics education independently.

Also, the praxeological lens allow us to see how different ambitions concerning the integration between programming and mathematics can look. Some teachers see programming as a natural part of mathematics. Others see programming as related to digital literacy and not really related to mathematics (Kilhamn & Rolandsson, 2021).

Concluding discussion

In conclusion, integrating programming as a tool for learning mathematics is a complex endeavor with both potentials and significant challenges. While the idea of using programming to enrich mathematical understanding is appealing, practical implementation often sees mathematics itself sidelined in favor of computational skills. Teaching programming within mathematics requires substantial instructional time, a high level of teacher expertise, and a nuanced understanding of what students learn through such activities. Providing students with pre-made programs to tinker with, rather than starting from scratch, may offer a more feasible approach to introducing computational thinking (CT) without overwhelming the mathematical focus. But this suggestion is rather speculative and more research is needed tied specifically to the educational systems and choices made in the different countries (Helenius & Misfeldt 2021).

From a CT perspective, initial experiments with programming in mathematics have shown promise. However, translating these experiments into genuine mathematical learning remains elusive. Effective integration calls for teaching approaches rooted in tinkering and exploration rather than beginning with coding from a blank slate. Designing tasks that genuinely support mathematical learning through programming is possible but challenging, often requiring specialized expertise that teachers alone may not have.

Reflecting on these challenges, the vision of using programming to teach mathematics is more complex in practice than in theory. While engaging for students, programming within mathematics raises unresolved questions about its core purpose and the boundaries of mathematical learning. Stakeholders across educational and policy spectrums continue to debate the role of programming in mathematics, prompting ongoing discussions about what constitutes the essence of mathematics education and whether it should include programming.

As this field evolves, there is a clear need for empirical research to understand better the difficulties encountered and successes achieved in various contexts. Shared insights and evaluations from different countries would provide valuable guidance for future efforts.

Acknowledgment

This work is supported by a Novo Nordisk Foundation grant "Programming, computational thinking and mathematical digital competencies" Grant number 0058651. NFF grant 23OC0084733 Svenska Vetenskapsrådet grant number 2018-03865, and "videncenter for teknologiforståelse" Novo Nordisk Foundation NNF23SA0082894 VILLUM foundation project 54607 and Lundbeck Foundation R419-2022-1261.

References

Bocconi, S., Chioccariello, A., Kampylis, P., Dagienė, V., Wastiau, P., Engelhardt, K., Earp, J., Horvath, M., Jasutė, E., Malagoli, C., Masiulionytė-Dagienė, V., & Stupurienė, G. (2022). *Reviewing Computational Thinking in Compulsory Education: State of play and practices from computing education*. Publications Office of the European Union, Luxembourg. https://doi.org/10.2760/126955

Bråting, K., & Kilhamn, C. (2021). Exploring the intersection of algebraic and computational thinking. *Mathematical Thinking and Learning*, 23(2), 170–185. https://doi.org/10.1080/10986065.2020.1779012

Børne- og undervisningsministeriet (2018). Kontraktbilag 1: Kravspecifikation. Forsøg med teknologiforståelse i folkeskolens obligatoriske undervisning. http://docplayer.dk/149137888-Kontraktbilag-1-kravspecifikation-forsoeg-med-teknologiforstaaelse-i-folkeskolens-obligatoriske-undervisning.htmlU

Børne- og undervisningsministeriet (2020). *Midtvejsevaluering: Forsøg med teknologiforståelse i folkeskolens obligatorisk undervisning.* Børne- og Undervisningsministeriet.

Chevallard, Y. (2019). Introducing the anthropological theory of the didactic an attempt at a principled approach. *Hiroshima journal of mathematics education*, 12, 71-114.

Elicer, R., Tamborg, A., Bråting, K., & Kilhamn, C. (2023) Comparing the integration of programming and computational thinking into Danish and Swedish elementary mathematics curriculum resources. *LUMAT* 11(3), 77–103. doi.org/10.31129/LUMAT.11.3.1940

Kallia, M., van Borkulo, S. P., Drijvers, P., Barendsen, E., & Tolboom, J. (2021). Characterising computational thinking in mathematics education: A literature-informed Delphi study. *Research in mathematics education*, *23*(2), 159-187.

Kilhamn, C., & Rolandsson, L. (2021). Utmaningar och möjligheter. In K. Bråting, L. Rolandsson & C. Kilhamn (Eds.), *Programmering i skolmatematiken – möjligheter och utmaningar* (pp. 178–194). Studentlitteratur.

Helenius, O., & Misfeldt, M. (2021). Programmeringens väg in i skolan: en jämförelse mellan Danmark och Sverige. I K. Bråting, C. Kilhamn, & L. Rolandsson (red.), *Programmering i skolmatematiken: möjligheter och utmaningar* (1 udg., s. 39-56). Studentlitteratur.

Misfeldt, M., & Ejsing-Duun, S. (2015). Learning Mathematics through Programming: An Instrumental Approach to Potentials and Pitfalls. In K. Krainer, & N. Vondrová (Eds.), *CERME9* (pp. 2524-2530). Charles University in Prague, Faculty of Education and ERME.

Jankvist, U. T., Misfeldt, M., Geraniou, E., Aguilar, M. S., & Baccaglini-Frank, A. (2023). Towards a technocritical mathematics education. *Nordic Studies in Mathematics Education*, *28*(3-4).

Skolverket (2018). Läroplan för grundskolan, förskoleklassen och fritidshemmet [Curriculum for primary school, pre-school and after-school classes]. https://www.skolverket.se/download/18.6bfaca41169863e6a65d48d/1553968042333/pdf3975.pdf

Tamborg, A. L. (2022). A solution to what? Aims and means of implementing informatics-related subjects in Sweden, Denmark, and England. *Acta Didactica Norden*, 16(4).

Tamborg, A. L., Nøhr, L., Løkkegaard, E. B., & Misfeldt, M. (2023). Computational thinking in educational policy – the relationship between goals and practices. I C. Damşa, & A. Barany (red*.), Advances in Quantitative Ethnography* (s. 299-213). Springer. Communications in Computer and Information Science, 1785 https://doi.org/10.1007/978-3-031-31726-2_21

Weintrop, D., Beheshti, E., Horn, M., Orton, K., Jona, K., Trouille, L., & Wilensky, U. (2016). Defining computational thinking for mathematics and science classrooms. *Journal of science education and technology*, *25*, 127-147.

Wing, J. M. (2006). Computational thinking. *Communications of the ACM*, *49*(3), 33-35.